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Featured abstracts

On a class of three-dimensional dynamic problems of layered shells with incomplete contact between some layers

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Keywords: asymptotic method, layered shell, inclinometer, earthquakes prediction

Plates and shells have the property that one of their dimensions (thickness) is significantly less than the tangential dimensions. This property allows moving to dimensionless forms of the equilibrium (movement) and state equations (deformations-stresses, connections) of the elasticity theory, introduce a small geometric parameter and apply the expansion method with respect to this small parameter. Yet, it turned out that the obtained new system is singularly perturbed with respect to the small parameter and it is impossible to apply the classical expansion method with respect to the small parameter. Here each unknown quantity has got its order with respect to the small parameter. These orders strongly depend on the nature of the boundary-value problem under consideration and the determination of their mutually not contradicting values is one of the most difficult problems of the method, which has been overcome in the best way in monograph [1].

Taking as a basis the geological structure (layer thicknesses, Young and shear modules, Poisson's ratio, density, thermal expansion coefficient) of the corresponding area of the net of inclinometers 3D dynamic problems for layered shells will be solved. By monitoring the change in the stress-strain state of the layered shell over time, according to the measuring equipment readings, there will be moments before earthquake when the strength of the contact surface between some layers of the slab will be violated, resulting in a rupture and the contact will become incomplete [2].

3D dynamic problems for three-layered orthotropic elastic shells are considered, with free upper face and prescribed displacements between the first and second layers the conditions of ideal contact and between the second and third layers assumed in incomplete contact. A long-wave asymptotic solution has been constructed, and the thickness resonances have been determined. The obtained results may find further applications in the evaluation of parameters of earthquakes.

References

[1] Aghalovyan L. Asymptotic Theory of Anisotropic Plates and Shells. *Singapore-London. World Scientific Publishing.* 376p (2015).

[2] Kasahara K. Earthquake mechanics. *Cambridge University Press.* 1981. 261p (1981).

On some methods analyzing reinforced materials and structures

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Keywords: Reinforced materials, Laminate theories, Particle reinforcement.

With respect to the demands of lightweight structural engineering classical structural materials are more and more substituted by fibre and particle reinforced materials in advanced, but also in ordinary structures. The aim of lightweight structural engineering is to save raw materials, costs and energy in the manufacture, use and recycling of a product. Especially with moving masses (road and rail vehicles, elevators, robot arms, machine components, etc.), lightweight structures can reduce operating costs or increase the payload.

Laminate and sandwich structures are typical lightweight elements with rapidly expanding application in various industrial fields. In the past, these structures were used primarily in aircraft and aerospace industries. Now, they have also found application in civil and mechanical engineering, in the automotive industry, in shipbuilding, the sport goods industries, etc. The main directions in modelling laminate and sandwich structures are discussed, for example, in [1].

Recently, polymers reinforced with short glass fibers (with a length of 0.1-1 mm, a diameter that, as a rule, does not exceed one tenth of their length, and a volume content of fibers of 15-45%) have been widely used in many fields of modern engineering. A great quantity of thin-walled structural elements operating under loads are manufactured by injection molding. Some elements of modelling these materials are discussed in [2].

For sufficiently high loading every material suffers some sort of gradual or abrupt mechanical failure (damage or plastic collapse). Its specific form depends on the material internal atomistic, molecular and/or topological structure. Composite materials, by definition, involve at least two distinct materials (phases) which increases the number of various failure possibilities. Further complication related to failure of composite materials is associated with presence of interphases. The interphase properties depend on many factors, including the physical and chemical properties of the phases as well as on processes used in composite manufacturing. For example, in [3] models of these structures are discussed.

In the paper a personal view of the author on these problems is given and advantages and disadvantages are presented.

References

[1] Altenbach, H., Altenbach, J., Kissing, W. Mechanics of Composite Structural Elements (2nd edition), Springer Nature Singapore Pte Ltd. (2018).

[2] Altenbach, H., Naumenko, K., L'vov, G.I., Pilipenko, S.N. "Numerical estimation of the elastic properties of thin-walled structures manufactured from short-fiber-reinforced thermoplastics", Mechanics of Composite Materials, 39(3), 221-234 (2003).

[3] Nazarenko, L., Stolarski, H., Altenbach, H. "Modeling cylindrical inhomogeneity of finite length with Steigmann–Ogden interface", Technologies, 8(78), 15 pages (2020).

Free vibration and flutter of a multi-component beam in a supersonic flow

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Keywords: Galerkin's Method; unit step function; Dirac's delta function; doublet function; finite element method.

Every physical phenomenon can be described via differential equations, but sometimes the problem addressed it is difficult to be solved by a classic analytical approach. This is the reason why many scientists have developed a wide range of methods to numerically solve differential equations; one of them is the Galerkin method.

We are interested in extending the range of validity of the Galerkin method to a stepped structure, whose cross-section and material parameters are not constant along the whole structure.

The aim of this study is to demonstrate that describing the discontinuities via generalized functions is the best way to implement the Galerkin method. Specifically, we investigate two cases of dynamic problem, i.e., free vibration and dynamic stability, namely the flutter; for both cases we compare two different implementations of the Galerkin method: (i) the straightforward implementation, well known in the literature, and (ii) the modified implementation: our proposal. The *straightforward* Galerkin method identifies the procedure employed to integrate the differential equation along each step; at the end of this procedure, all the partial results are summed. Conversely, the *modified* Galerkin method, after we have defined the differential problem via generalized functions (the Heaviside function and its derivatives), integrates the problem over the whole domain.

The first section is dedicated to free vibration problems, while the second to the flutter problem. In the whole study, we investigate a uniform, simply-supported Euler-Bernoulli beam in three different material cases. The first and the second material cases are both homogeneous; more in detail, the first is characterized by elastic modulus and mass density greater than the second one. The third and final case is the stepped one: we subdivide the beam length in three equal portions following the pattern Strong-Weak-Strong material. We apply, then, both the straightforward and the modified Galerkin method and we compare the obtained results with the exact, analytical solution regarding the free vibration problem and with a finite element solution, coded by us, for the flutter section.

To conclude, this study demonstrates that generalized functions are a valid tool to describe concentrated discontinuities; in fact, the modified Galerkin method reaches convergence by increasing the number of the terms in the Galerkin expansion, while the straightforward method tends to a wrong solution even if we increase the terms in the expansion.

References

[1] Bolotin, V.V., *Nonconservative Problems of the Theory of Elastic Stability*, The Macmillan Company, New York, 1963.

[2] Dowell, E.H., and Bendiksen, O., "Panel Flutter," *Encyclopedia of Aerospace Engineering*, 2010.

[3] Marzani, A., Mazzotti, M., Viola, E., Vittori, P., and Elishakoff, I., "FEM Formulation for Dynamic Instability of Fluid-Conveying Pipe on Nonuniform Elastic Foundation," *Mechanics Based Design of Structures and Machines*, Vol. 40, No. 1, 2012, 83-95.

Homogenization of transport properties of densely packed, high-contrast fibre composites: analytical solution of cell problem

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Keywords: Densely packed composite, high-contrast components, fibre composites, homogenization, lubrication approximation, Padé approximants.

Paper is devoted to calculation of the effective stationary and non-stationary heat conductivity for fibre composite materials. Various variants of lattices (square [1, 3], hexagonal [2, 4]) and inclusions (circle [1], square [3], curvilinear rhombic [2]) are under consideration. Different conditions of interphase contact including a thin interlayer at the boundary of matrix and inclusion are investigated [1, 2].

The multiscale asymptotic procedure is used, which separates the local and global BVP of the original problem. Original problem for multiply-connected domain is reduced to the sequence of boundary value problems in simply-connected domains. For non-stationary problem time variable excludes from the original boundary value problem using Laplace transform.

It has been shown that for densely packed, high-contrast fiber composites, it is possible to obtain an analytical solution to the problem on a cell using lubrication approximation [5] and Padé approximants.

References

[1] Andrianov, I.V., Awrejcewicz, J., Starushenko, G.A., „Asymptotic models and transport properties of densely packed, high-contrast fibre composites. Part I: Square lattice of circular inclusions”, *Compos. Struct.*, 179, 617-627 (2017).

[2] Andrianov, I.V., Awrejcewicz, J., Starushenko, G.A., „Asymptotic models for transport properties of densely packed, high-contrast fibre composites. Part II: Square lattices of rhombic inclusions and hexagonal lattices of circular inclusions”, *Compos. Struct.*, 180, 351-359 (2017).

[3] Andrianov, I.V., Kalamkarov, A.L., Starushenko, G.A., „Analytical expressions for effective thermal conductivity of composite materials with inclusions of square cross-section”, *Compos. B. Eng.*, 50, 44-53 (2013).

[4] Kalamkarov, A.L., Andrianov, I.V., Pacheco, P.M.C.L., Savi, M.A., Starushenko, G.A., „Asymptotic analysis of fiber-reinforced composites of hexagonal structure”, *J. Multiscale Model.*, 7(3), 1650006 (2016).

[5] Christensen, R.M., *Mechanics of Composite Materials*. Dover Publisher: Mineola, NY (2005).

Free vibrations of bi-stable pressurized FG plate-type MEMS

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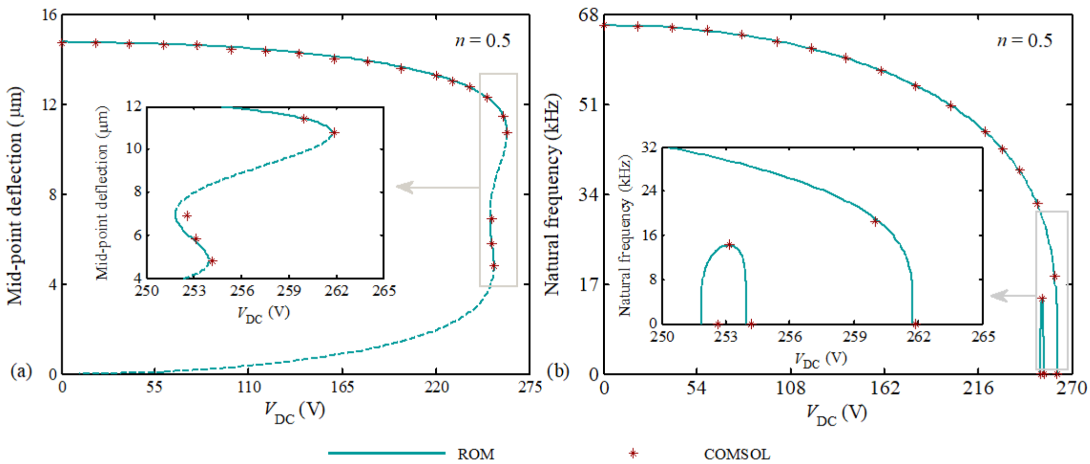
Keywords: Bi-stability, Functionally Graded Materials, Micro-plates.

Recently, it has been shown that besides initially curved micro-plates, pressurized flat ones can also experience bi-stable behavior that enjoys many potential applications in designing high sensitive MEMS sensors [1]. In the sequence of ref. [1], the present work aims to investigate free vibrations of pressurized electrically actuated thin micro-plates made of FGM. To this end, the geometric nonlinear in-plane and out-of-plane governing equations of motion are obtained using the Hamilton principle and reduced to an initial value problem associated with the first transversal eigenmode of the structure through a multi-term Galerkin projection method. Having the reduced equation of motion, the eigenvalue equation governing the free vibrations of the micro-plate around its equilibrium configuration is then obtained. Afterward, the oscillatory behavior of pressurized plate-type MEMS is studied. It has been observed that the fundamental frequency of the system suddenly drops to zero when the micro-plate faces the limit points in its equilibrium path. The present findings are compared and successfully validated by 3D FE simulations carried out in COMSOL Multiphysics commercial software for a Silicon-Copper graded micro-plate with dimensions as same as the case studied in ref. [1], the power-law index is set to $n = 0.5$, and the micro-plate is assumed to be subjected to 3kPa differential pressure in the opposite direction of the electrical attraction.



Acknowledgments

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References

[1] A.R. Askari, "Bi-stability of pressurized electrically actuated flat micro-plates", Int. J. Solids Struct., 178-179, 167 - 179 (2019).

Growth and remodelling in the mechanics of human brain organoids

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Keywords: Organoids, Growth, Remodelling, Instabilities.

Organoids are prototypes of human organs derived from cultured human stem cells. They provide a reliable and accurate experimental model to study the physical mechanisms underlying the early developmental stages of human organs and in particular, the early morphogenesis of the cortex. Here, we propose a mathematical model to elucidate the role played by two mechanisms that have been experimentally proven to be crucial in shaping human brain organoids: the contraction of the inner core of the organoid and the micro-structural remodeling of its outer cortex. Our results show that both mechanisms are crucial for the final shape of the organoid and that perturbing those mechanisms can lead to pathological morphologies which are reminiscent of those associated with lissencephaly (smooth brain).

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References

[1] V. Balbi, M. Destrade, A. Goriely. "Mechanics of human brain organoids", Phys. Rev. E, 101-, 022403 (2020).

Metamaterials for applications at extreme deformation

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Keywords: Homogenization, Lattice materials, Ellipticity loss, Shear bands.

Two-dimensional lattice materials made up of elastic rods, axially pre-loaded and subject to incremental deformation (involving normal and shear forces and bending moment) are treated with homogenization theory to find the equivalent prestressed elastic solid, as in [1,2]. This solid may be subject to loss of ellipticity (representing shear band formation) or to microinstabilities (only visible as a structural response of the lattice). It is shown [3] that the lattice would be ideal not only to theoretically analyze instabilities, but also to practically realize the porous architected materials which are preconized to yield extreme mechanical properties such as foldability, channelled response, and surface effects [4].

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References

[1] Triantafyllidis, N., Maker., B.N., "On the Comparison Between Microscopic and Macroscopic Instability Mechanisms in a Class of Fiber-Reinforced Composites". J. Appl. Mech., 52, 794-800 (1985).

[2] Ponte Castañeda, P., "The Effective Mechanical Properties of Nonlinear Isotropic Composites" J. Mech. Phys. Solids 39, 45–71 (1991).

[3] Bordiga, G., Cabras, L., Piccolroaz, A., Bigoni, D., "Dynamics of prestressed elastic lattices: Homogenization, instabilities, and strain localization". J. Mech. Phys. Solids, 146, 104198 (2021).

[4] Kochmann, D.M., Bertoldi, K., "Exploiting Microstructural Instabilities in Solids and Structures: From Metamaterials to Structural Transitions". Appl. Mech. Rev., 69, (2017).

Dynamics of bi-laminated plates with soft viscoelastic core

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Keywords: Stratified plates ; Viscoelasticity ; Green functions ; Bending waves ; Temperature

This article deals with the dynamics of bilaminated plate with soft viscoelastic core. The study is based on the bi-torsor description of Highly Contrasted Stratified (HCS) plate derived by asymptotic approach [1]. The model accounts for the overall, and inner, shear forces and bending moments $\{(\underline{\mathcal{T}}, \underline{\underline{\mathcal{M}}}) ; (\underline{T}, \underline{\underline{M}})\}$, and for the additional kinematic descriptor constituted by the internal sliding. The constitutive laws that relates linearly (i) $\underline{\underline{\mathcal{M}}}$ to the overall rotation vector $\underline{\alpha}$, (ii) $\underline{\underline{M}}$ to the gradient of deflection $\underline{\nabla} w$, and (iii) $\underline{T}$ to the sliding vector $\underline{\delta}$, are explicitly determined from the mechanical properties and the geometry of the layers. The equations governing the out-of-plane behavior of the HCS plate loaded by normal surface force *f* reads [1] :

$$\mathrm{div}(\underline{\mathcal{T}}) + f = 0 \quad ; \quad \underline{\mathcal{T}} = -\underline{\mathrm{div}}(\underline{\underline{M}}) + \underline{T} \quad ; \quad \underline{T} = -\underline{\mathrm{div}}(\underline{\underline{\mathcal{M}}}) \tag{1}$$

This approach enables to derive exact analytical formulations that applies either for elastic plate or to plates with visco-elastic constituents provided that they are expressed in the Fourier domain. Due to the viscoelastic and thermo-sensible behavior of the core an energy loss occurs by viscous internal dissipation and the dynamic behavior is significantly influenced by the temperature. The paper addresses these two delicate issues by:

- analyzing the feature of the three bending waves, with explicit determination of the damping,
- establishing the analytical expression of the Green function of elastic and viscoelastic HCS plates,
- performing a "quasi"-exact modal analysis of viscoelastic HCS-plates,
- quantifying the effect of temperature on the eigen modes, in terms of frequency and damping.

These developments will be illustrated in the case of laminated glass with PolyVinylButiral inter-layer, [2]. Finally, vibrational experiments performed on such glass plate at different temperatures will be shown to be in good agreement with the theoretical modeling.

References

[1] Boutin C., Viverge K. Generalized plate model for highly contrasted laminates, European Journal of Mechanics - A/Solids, 55 (2016) 149-166

[2] Boutin C., Viverge K., Hans S. Dynamics of contrasted stratified elastic and viscoelastic plates - application to laminated glass, Composites Part B: Engineering, 212 (2021)

[3] Foraboschi P. Layered plate with discontinuous connection: Exact mathematical model, Composites: Part B 47 (2013) 365–378,

[4] Naumenko K., Eremeyev V.A., A layer-wise theory for laminated glass and photovoltaic panels, Composites Structures 112 (2014) 283–291

[5] Berdichevsky V.L. An asymptotic theory of sandwich plates, Int. Journal of Engineering Science, 48, (2010) 383-404

Simulation of interaction of Rayleigh waves with complex shape near-surface barriers utilizing linear and nonlinear material models

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Keywords: Rayleigh waves, seismic protection, metamaterials.

The paper will present simulation results for interaction of Rayleigh surface waves with the near-surface inclusions of complex shape. The effect of material constitutive model utilized for material representing inclusion will also be studied. It will be demonstrated that inclusion of additional geometric features supplementing standard rectangular barrier geometry can significantly affect interaction with oncoming Rayleigh waves and, hence can have a great impact on the barrier protective properties. The effect of material model utilized for the barrier material is also studied. It will be shown that accounting for plasticity by introduction of Coulomb-Mohr material constitutive model can significantly affect interaction with oncoming Rayleigh surface waves. The Coulomb-Mohr material model is applicable to simulate the behaviour of granular metamaterials that are planned to be utilized to fill the protective barriers. It is demonstrated that granular metamaterials have a potential for significant reduction of vibrations caused by oncoming Rayleigh waves, compared to materials with linear elastic behaviour. The results of the presented study can be utilized for design of optimal shape and material to be used for seismic protective barrier.

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Stochastic homogenisation of high-contrast media

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Keywords: Homogenisation, Spectral Theory, High-contrast Random Composites.

We study the homogenisation problem for elliptic operators of the form $\mathcal{A}_\varepsilon = -\nabla A^\varepsilon \nabla$ with high-contrast random coefficients A^ε. In particular, we are interested in the behaviour of their spectra. We assume that on one of the components of the composite the coefficients A^ε are "of order one", the complimentary "soft" component consists of randomly distributed inclusions, whose size and spacing are of order $\varepsilon \ll 1$, and the values of A^ε on the inclusions are of order ε^2.

Our interest in high-contrast homogenisation problems is motivated by the band-gap structure of their spectra. From a mathematically rigorous perspective, these were first analysed in [1], [2] in the periodic setting.

Unlike the periodic high-contrast operators, whose spectra are described by similar multiscale arguments for a bounded domain and for the whole space, in the stochastic case the situation is fundamentally different. In the bounded domain setting our findings and the basic techniques are similar to those of the periodic case. In the whole space setting, however, the spectrum $\mathrm{Sp}(\mathcal{A}^{\mathrm{hom}})$ is, in general, a proper subset of the Hausdorff limit of $\mathrm{Sp}(\mathcal{A}^\varepsilon)$. In fact, the case when the spectrum of $\mathcal{A}^\varepsilon$ occupies the whole positive half-line is not uncommon. This additional spectrum, which does not appear in the bounded-domain setting, is attributed to the stochastic nature of the problem. We provide a comprehensive analysis of the homogenised operator $\mathcal{A}^{\mathrm{hom}}$ and its spectrum, characterise the limit of $\mathrm{Sp}(\mathcal{A}^\varepsilon)$, and study the relation between $\lim_{\varepsilon\to 0}\mathrm{Sp}(\mathcal{A}^\varepsilon)$ and $\mathrm{Sp}(\mathcal{A}^{\mathrm{hom}})$ under some lenient assumptions on the regularity of the inclusions.

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References

[1] Zhikov, V.V., "On an extension of the method of two-scale convergence and its applications", (Russian) Mat. Sb., 191(7), 31-72; translation in Sb. Math., 191(7-8), 973-1014 (2000).

[2] Zhikov, V.V., "Gaps in the spectrum of some elliptic operators in divergent form with periodic coefficients", (Russian) Algebra i Analiz, 16 (5), 34-58; translation in St. Petersburg Math. J. 16(5), 773-790 (2004).

Sharp operator-norm asymptotics for thin elastic plates with rapidly oscillating periodic properties

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Keywords: Homogenisation, dimension reduction, effective properties, asymptotics, Korn inequalities, elastic plates.

We analyse a system of partial differential equations describing the behaviour of an elastic plate with periodic moduli in the two planar directions, in the asymptotic regime when the period and the plate thickness are of the same order. Assuming that the displacement gradients of the points of the plate are small enough for the equations of linearised elasticity to be a suitable approximation of the material response, such as the case in e.g. acoustic wave propagation, we derive a class of "hybrid", homogenisation dimension-reduction, norm-resolvent estimates for the plate, under different energy scalings with respect to the plate thickness.

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References

1] K. Cherednichenko, I. Velcic. Sharp operator-norm asymptotics for thin elastic plates with´ rapidly oscillating periodic properties, 36 pp., arXiv: 1802.02639.

Can one use averaging when modelling toughness heterogeneity in hydraulic fracture?

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We discuss the various approaches used to capture heterogeneity within a reservoir undergoing hydraulic fracturing treatment and their implications for modelling fracture propagation. In highly laminated reservoirs with soft and/or weak layers, capturing heterogeneity at an appropriate resolution is the key for successful prediction of fracture growth and other crucial treatment parameters [1,2].

Typically, when using computational methods such as the Finite Element Method, the well log and petrophysical data obtained from various measurements and observations are upscaled and/or "homogenized" to the element size. From all in-situ parameters, toughness is one of the most delicate physical parameters to handle, as the effectiveness of homogenization techniques utilized are questionable [3,4]. Compared to other types of fracture evolution, hydraulic fracturing is probably the most stable crack propagation process and thus, fortunately, some estimates can be provided.

We estimate errors introduced by various strategies to incorporate heterogeneous fracture toughness into numerical modelling [5,6]. We restrict ourselves to periodic distributions and consider only the KGD model without leak-off, allowing us to easily handle different regimes (toughness/viscosity). For the simulations, we use an extremely effective (in house-built) time – space adaptive solver incorporating ideas reported in [7]. The solver is capable of computing rather arbitrary distribution(s) of the toughness.

We propose and analyse the notion of an "effective" toughness and show that it is a process dependent variable. Comparisons with the average toughness over the domain are performed. Finally, we provide some recommendations on implementing the defined measure for numerically modelling hydraulic fracture.

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References

[1] Gaillot, P. 2020. High Resolution Petrophysics and Geomechanics Workflow – One Key to Optimal Development of Unconventional Reservoirs. Paper presented at the SPE/AAPG/SEG Unconventional Resources Technology Conference, Virtual, July 2020. URTeC-2020-2180

[2] Richards, G.J., Bere, A.T., Roberts, D.T., Thomas, S.D., Mutlu, O. and Gaillot, P.J. 2020. Hydraulic fracturing in heterogenous reservoirs; modelling at petrophysical vs. geomechanical resolution. In Proc the 54th US Rock Mechanics/Geomechanics Symposium, Golden, Colorado, USA, July 2020, eds. C.S. Desai et al, 163–168. Rotterdam: Balkema.

[3] Caiulo, A. and M. Kachanov, 2010. On absence of quantitative correlations between strength and stiffness in microcracking materials. Int. J. Fract., 164:155–158

[4] Kachanov, M., 1994. On concept of damage in creep and in the brittle-elastic range. Int. J. Dam. Mech., 3.

[5] Dontsov, E.V., and R. Suarez-Rivera. 2021. Representation of high-resolution rock properties on a coarser grid for hydraulic fracture modeling. J. Petrol. Sci. Eng., 198, 108144. 6.

[6] Da Fies, G., Dutko, M. and Mishuris, G. 2021. Remarks on dealing with toughness heterogeneity in modelling of hydraulic fracture. Paper presented at the ARMA-21 Symposium, Virtual, Texas, USA, April 2021. ARMA-21-2010.

[7] Wrobel, M. and Mishuris, G. 2015. Hydraulic fracture revisited: Particle velocity based simulation. Int. J. Eng. Sci., 94: 23-58

Shear band patterns arising from rigid line inclusion distributions via boundary element technique

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Keywords: High-contrast composites, rigid platelets, failure mechanisms.

Similarly to cracks, stress concentrations may arise at the tips of rigid line inclusions. These stress concentrations are analytically predicted [2] and experimentally [5] proven to be square root singularities when the matrix material is linear elastic. Moreover, in the case when the matrix is uniformely prestressed close to the condition of loss ellipticity, the shear band pattern at failure is strongly affected by the presence of a rigid line inclusion [3]-[4].

Based on the Green's function for incremental nonlinear elasticity [1], boundary integral equations are formulated in the presence of a generic distribution of N rigid line inclusions. The numerical solution is obtained through boundary element technique with varying the inclusion distribution geometry. The distribution influence on the shear band pattern is discussed.

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References

[1] Bigoni, D., Capuani, D. "Green's function for incremental nonlinear elasticity: shear bands and boundary integral formulation", J. Mech. Phys. Solids 50, 471–500 (2002).

[2] Chen, Y.Z "Singular behaviour at fixed rigid line tip in plane elasticity", Eng. Frac. Mech., 25(1):11–16, (1986).

[3] Dal Corso, F., Bigoni, D. "The interactions between shear bands and rigid lamellar inclusions in a ductile metal matrix", Proc. R. Soc. Lond. A 465, 143-163 (2009).

[4] Misra, S., Mandal, N. "Localization of plastic zones in rocks around rigid inclusions: Insights from experimental and theoretical models", J. Geoph. Res.: Sol. Earth, 112(B9), (2007).

[5] Noselli, G., Dal Corso, F. and Bigoni, D. "The stress intensity near a stiffener disclosed by photoelasticity", Int. J. Fract., 166, 91–103 (2010)

Free, forced, and random vibrations of a beam composed of highly contrasting materials

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Keywords: Bi-material beam; Highly contracting materials; Random vibration; Cutoff frequency

This study deals with free, forced undamped vibration and the random, damped vibration of an inhomogeneous beam composed of two materials with significantly different elastic moduli and material densities. Exact solution is obtained for natural frequencies and the mode shapes using Krylov-Duncan functions, along with the comparison with the results furnished by the Galerkin's method. Random vibrations of the bi-material beams are studied for the first time in the literature for the case of proportional damping. Novel element of this study consists in the unintuitive finding that the response of the bi-material beam may considerably exceed the responses of its homogeneous counterparts, composed of a single material. This finding appears of design significance, suggesting that the natural frequencies of the bi-material beam must be made so as to lie above the cutoff frequency of broad-band cut white noise.

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References

[1] Z. Qiao, Z. Weihong, Z. Jihong, G. Tong, Layout optimization of multi-component structures under static loads and random excitations, Engineering Structures 43 (2012) 120–128, doi:10.1016/j.engstruct.2012.05.013.

[2] Hussein, M.I., Leamy, M.J. and Ruzzene, M., Dynamics of phononic materials and structures: historical origins, recent progress, and future outlook, Applied Mechanics Reviews, 66(4), (2014) 040802, 10.1115/1.4026911.

[3] Mace, B.R., Discussion of "Dynamics of phononic materials and structures: historical origins, recent progress and future outlook" (Hussein, M. I., Leamy, M.J., Ruzzene, M., 2014, ASME Appl. Mech. Rev., 66 (4), p. 040802). Appl Mech Rev, 66(4), 045502, 10.1115/1.4027723.

[4] Hussein M.I., M.J. Leamy, M. Ruzzene, Closure to "Discussion of 'Dynamics of Phononic Materials and Structures: historical Origins, Recent Progress, and Future Outlook'", Appl Mech Rev 66 (4) (2014) 046002, doi:10.1115/1.4027723.

[5] M. Aßmus, S. Bergmann, K. Naumenko, H. Altenbach, Mechanical behavior of photovoltaic composite structures: a parameter study on the influence of geometric dimensions and material properties under static loading, Composites Communications 5 (2017) 23–26, doi:10.1016/j.coco.2017.06.003.

[6] K. Viverge, C. Boutin, F. Sallet, Model of highly contrasted plates versus experiments on laminated glass, Int J Solids Struct 102 (2016) 238–258, doi:10.1016/j.ijsolstr.2016.09.035.

[7] Y. Qin, X. Wang, Z.L. Wang, Microfibre–nanowire hybrid structure for energy scavenging, Nature 451 (2008) 809–813, doi:10.1038/nature06601.

[8] M. Ruzzene, A. Baz, Attenuation and localization of wave propagation in periodic rods using shape memory inserts, Smart Materials and Structures 9 (6) (2000) 805–816, doi:10.1088/0964-1726/9/6/310.

Frequency spectra and stop-band optimisation of generalised canonical quasicrystalline phononic waveguides

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Keywords: Floquet-Bloch technique, metamaterials, composite materials.

The dynamic properties of periodic generalised two-phase quasicrystalline phononic rods [1] are studied through the method of the toroidal manifold. The method allows all stop and pass bands featuring the frequency spectrum to be represented in a compact form with a frequency dependent flow line on the surface describing their ordered sequence. The flow lines on the torus can be either closed or open: in the former case, (i) the frequency spectrum is periodic and the elementary cell corresponds to a *canonical configuration* [2], (ii) the stop-band density depends on the lengths of the two phases; in the latter, the flow lines cover the torus and the stop-band density is independent of those lengths. It is then shown how the proposed compact description of the spectrum can be exploited to optimise the layout of the elementary cell in order to maximise the low-frequency stop band. The scaling property of the frequency spectrum, that is a distinctive feature of quasicrystalline generated phononic media, is rigorously introduced. The obtained results represent both a key to a better understanding of the properties of classical two-phase composite waveguides and an important advancement towards the realisation of composite quasicrystalline metamaterials.

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References

[1] L. Morini, M. Gei, Waves in one-dimensional quasicrystalline structures: dynamical trace mapping, scaling and self-similarity of the spectrum, J. Mech. Phys. Solids 119 (2018) 83-103.

[2] M. Gei, Z. Chen, F. Bosi, L. Morini, Phononic canonical quasicrystalline waveguides, Appl. Phys. Lett. 116 (2020) 241903.

Pattern formation on an elastic half-space coated by a periodic hard layer

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Keywords: surface wrinkling, buckling, periodic beam, Floquet theory.

Wrinkling is a common phenomenon in nature and it can give rise to various surface patterns which have a wide range of applications in biomedical, functional coatings, and flexible electronic devices [1, 2]. Wrinkling of a homogeneous elastic half-space coated by a homogeneous layer is one of the simplest prototypical problems that has been much studied in recent decades [3, 4]. With a view to uncovering novel patterns, Feng et al. [5] recently considered pattern formation on a hardened skin layer with a periodic stiffness distribution on a soft material. A variety of surface patterns were numerically simulated and confirmed experimentally. Limited analysis was also given in order to explain the mechanisms of pattern formation.

Our current study aims to provide a more comprehensive analysis of pattern formation on an elastic half-space coated by a periodic hard layer than in [5]. Our ultimate goal is to model both the layer and half-space using the exact theory of nonlinear elasticity, but as a first step, we consider the reduced model of an Euler beam with periodic material properties supported by a Winkler foundation. We use Floquet theory and transfer matrix method to derive the bifurcation condition, and analyse a number of asymptotic limits. Our asymptotic results are then compared with the experimental and numerical simulation results in [5].

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References

[1] Li, B, Cao, Y P, Feng, X Q, and Gao, H J, "Mechanics of morphological instabilities and surface wrinkling in soft materials: a review", Soft Matter, 8,5728-5745 (2012).

[2] Dimmock, R L, Wang, X L, Fu, Y B, El Haj, A J, and Yang, Y, "Biomedical applications of wrinkling polymers", Recent progress in materials, 2, 2001005 (2020).

[3] Bowden, N, Brittain, S, Evans, A G, Hutchinson, J W, and Whitesides, G M "Spontaneous formation of ordered structures in thin films of metals supported on an elastomeric polymer", Nature, 393, 146-149 (1998).

[4] Cai, Z X, and Fu, Y B, "On the imperfection sensitivity of a coated elastic half-space", Proc. R. Soc. Lond., A 455, 3285-3309 (1999).

[5] Wang J W, Li B, Cao Y P, Feng X Q, Gao H J, "Wrinkling micropatterns regulated by a hard skin layer with a periodic stiffness distribution on a soft material", Applied Physics Letters, 108, 021903 (2016).

Free vibrations of thin elastic composed cylindrical panel with free edges and with longitudinal section of material properties

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Keywords: boundary vibrations, eigenfrequencies, composite cylindrical panel.

It is known that depending on geometrical and mechanical parameters of a plate and cylindrical panel, a complex distribution picture of natural frequencies arises in plate and cylindrical panel with free edge [1]. With growing number of free edges of a cylindrical panel, this picture becomes increasingly complex [2]. Therefore, investigations into the edge resonance of composed cylindrical panels with free edges are among the most challenging problems in the theory of vibrations of plates and shells [1], [2], [3].

Using a system of equations corresponding to the classical theory of orthotropic cylindrical shells, free vibrations of thin elastic cylindrical panel, composed of two orthotropic cylindrical panels with different elastic properties, with full contact along the generatrices and with free edges are investigated. In order to calculate the natural frequencies and to identify the respective natural modes, the generalized Kantorovich-Vlasov method of reduction to ordinary differential equations is used. Dispersion equations for finding the natural frequencies of possible types of interfacial and boundary vibrations are derived. An asymptotic relation between the dispersion equations of the problem at hand and the analogous problem for a rectangular plate is established. An algorithm for separating possible interfacial and boundary vibrations is presented. As an example, the values of dimensionless characteristics of natural frequencies are derived for a composed cylindrical panel.

References

[1] Wilde M.V., Kaplunov Yu. D., and Kassovich L. Yu., Boundary and Interface Resonant Phenomena in Elastic Bodies (in Russian), Moscow, Fizmatlit (2010).

[2] Ghulghazaryan G.R., Ghulghazaryan L.G. and Kudish I.I. Free vibrations of thin elastic orthotropic cylindrical panel with free edges. Mechanics of Composite Materials, Vol. 55, No 5, November,(2019).

[3] Ghulghazaryan G.R. Free vibrations of thin elastic orthotropic cylindrical panel with hinge-mounted end. (in Russian) Izv.NAS of Armenii, Mech. 73(4), 29-47 (2020). Doi- http://doi.org/10.33018/73.4.3 .

Branched flow of flexural waves in random elastic plates

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Keywords: Flexural waves, Random media, Elastic plates of non-uniform thickness.

Waves travelling in random media with correlated disorder show the formation of branches or channels, which leads to emergence of random focusing events [1]. The phenomenon has been observed previously for non-dispersive waves in other physical contexts at many different length scales: from visible light [2] to tsunami waves [3]. Here we present the emergence of branched flow of dispersive flexural waves in elastic plates governed by the biharmonic equation. Simulations are carried out using finite element analysis and an analytical expression for the expected location of first caustic is obtained. Flow patterns connecting the locations of high amplitude are observed. The expected location of the first focusing event has an elegant scaling with respect to correlation length and the severity of randomness. This expected location is also independent of wavelength. The scaling relations derived here analytically for bending waves in elastic plates are consistent with our numerical simulations.

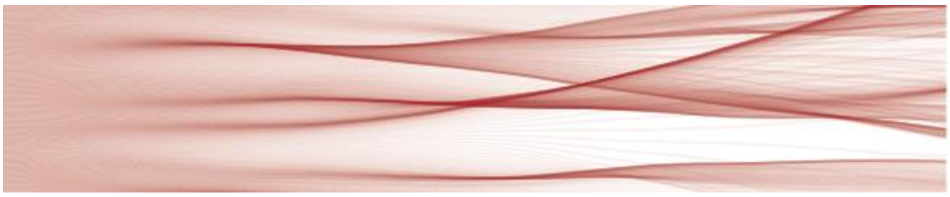

Figure 1: Ray simulations of waves supported by random elastic plates. Emergence of branching and random focusing can be seen.

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References

[1] Metzger, Jakob J., et al. "Universal statistics of branched flows." Physical Review Letters 105.2 (2010): 020601.

[2] Patsyk, Anatoly, et al. "Observation of branched flow of light." Nature 583.7814 (2020): 60-65.

[3] Degueldre, Henri, et al. "Random focusing of tsunami waves." Nature Physics 12.3 (2016): 259-262.

Uniform asymptotics for a family of degenerating variational problems with applications to error estimates in high-contrast homogenisation problems

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Keywords: Homogenisation, high-contrast, error estimates.

In recent years, various high-contrast elliptic PDEs have been proposed to model non-trivial physical phenomena in composite materials. One class of such PDE takes the form

$$-\mathrm{div} A_\varepsilon \nabla u_\varepsilon + \rho_\varepsilon u_\varepsilon = f, \tag{1}$$

where the coefficients are ε-periodic and may degenerate as ε tends to zero (e.g. ellipticity or boundedness is lost in the limit). The interest in studying degenerate models largely lies in the fact that the effective equations (e.g. the (two-scale) homogenised equations) for (1) possess 'novel' properties; i.e. properties that are not present in the homogenised equations for the (classical) non-degenerate setting. Such properties have been shown in various specific settings to be practically desirable.

This leads to a natural and, until now, largely open question of whether or not the original equations (1) possess the same desirable properties (as their limits) for finite ε (rather than just when ε tends to zero); or to put this question another way: can one establish a leading-order approximation, with respect to ε, for u_ε with error estimates uniform with respect to f (in various norms).

We study asymptotics of solutions to a family of degenerating variational problems that appear in the context of deriving effective equations for highly oscillatory degenerating linear PDE systems:

$$\begin{cases} \text{For each } \varepsilon > 0 \text{ and } \theta \in \Theta, \text{ find } u_{\varepsilon,\theta} \in H \text{ such that} \\ \varepsilon^{-2} a_\theta(u_{\varepsilon,\theta}, \phi) + b_\theta(u_{\varepsilon,\theta}, \phi) = \langle f, \phi \rangle, \quad \forall \phi \in H, \end{cases} \tag{2}$$

where $\Theta \subset \mathbb{R}^n$ is compact, H a complex Hilbert space, f a bounded linear functional on H, a_θ and b_θ are non-negative bounded sesquilinear forms such that $a_\theta + b_\theta$ form a family of uniformly equivalent inner products on H, and a_θ is Lipschitz-continuous in θ.

The formulation includes: partial high-contrast elliptic systems, i.e. (1) with $A_\varepsilon = a_1(\frac{\cdot}{\varepsilon}) + \varepsilon^2 a_0(\frac{\cdot}{\varepsilon})$ where $a_0 + a_1$ is positive and bounded (this model generalises the classical and doubleporosity model); (2) also includes Schrodinger's equation with a strong periodic magnetic field; differential-difference equations; homogenisation problems on periodic quantum graphs and their generalisations; problems in thin domains; problems with concentrated masses; and some higher order differential and pseudo-differential operators also fall into the framework (2).

A hierarchy of approximation results with error estimates is established under various assumptions, on the basic characteristics of the variational problems, observed in practical examples. Applications to spectral problems are considered and we provide approximations of the spectrum in terms of the spectrum of some effective 'bivariate' operator which generalises the two-scale homogenised operator found via two-scale convergence methods for highly oscillatory PDE. Our approach starts as the well-established spectral method, of classical homogenisation, but the techniques we develop do not rely on analytic perturbation theory and this allows us to readily consider a much wider class of degenerating problems including high-contrast homogenisation problems.

Asymptotic analysis of boundary value problems for high-contrast thin elastic laminates.

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Keywords: laminates, high-contrast, elastic, shear modes.

Slowly decaying boundary layers (with typical length scale much greater that the thickness) are a key feature of static behavior of high-contrast thin layered structures, see e.g. [1]. In parallel, extra low-frequency vibration modes are also observed in associated dynamic problems [2]. This subject finds various engineering applications, in particular, for photovoltaic panels and laminated glass [3, 4].

This talk is aimed at capturing both of the above mentioned features within a single long-wave low-frequency approximate setup. As an example, an anti-plane shear of a semi-infinite, asymmetric sandwich is considered [5]. First, it is shown that the exact dispersion relation allows a two-mode polynomial approximation, governing both the fundamental mode and the slowly varying evanescent mode transforming to the first harmonic. Then, the corresponding one-dimensional equations of motion are derived along with the asymptotic boundary conditions based on a nontrivial generalization of the classical Saint-Venant's principle.

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References

[1] Horgan, C.O., "Saint-Venant end effects for sandwich structures", *IV Int. Conf. Sandwich Constructions*, 1, 191-200, EMAS Publishing, UK (1998).

[2] Kaplunov, J., Prikazchikov, D.A., Prikazchikova, L.A. "The breakdown of continuum models for mechanical contacts", *Int. J. Solid Struct.*, 113, 169-179 (2017).

[3] Eisenträger, J., Naumenko, K., Altenbach, H., Meenen, J. "A user-defined finite element for laminated glass panels and photovoltaic modules based on a layer-wise theory", *Comp.Struct.*, 133, 265-277 (2015).

[4] Boutin, C., Viverge, K., Hans, S. "Dynamics of contrasted stratified elastic and viscoelastic plates – application to laminated glass", *Comp. Struct.B: Eng.*, 212, 108551 (2021).

[5] Kaplunov, J., Prikazchikova, L., Alkinidri, M. "Antiplane shear of an asymmetric sandwich plate", *Cont. Mech. Thermodyn.*, 33, 1247-1262 (2021).
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Norm-resolvent convergence to zero-range models with internal structure in models with strong inhomogeneities

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Keywords: High contrast medium, spectral asymptotics, norm-resolvent convergence, spectral theory.

In this talk, I will consider a prototype large-coupling transmission problem, posed on a bounded domain, containing a "low-index" inclusion located at a positive distance to the boundary. Mathematically, this is modelled by a "weighted" Laplacian $-a_{\pm}\Delta$, where $a_+ = 1$ (the weight on the containing domain) and $a_- \equiv a$ (the weight on the inclusion) is assumed to be large, $a_- \gg 1$. This is supplemented by the Neumann boundary condition on the outer boundary and "natural" continuity conditions on the interface (i.e., the inclusion boundary). A formal asymptotic argument suggests that eigenvalues of this operator should converge (as $a \to +\infty$) to those of the so-called "electrostatic problem" discussed in [2] and references therein (although the existing results do not quite yield spectral convergence). The operator-theoretic approach we use in [1], based on our analysis of critical-contrast periodic composites as presented in [3], allows one to improve these results in two respects: a)our estimates are of the operator-norm resolvent type, implying, in particular, the uniform convergence of the associated spectra in any compact set of $\mathbb{C}$; b) our estimates are uniform with respect to the "contrast" parameter a and are order-sharp, i.e. the rate of convergence in terms of $a \to 0$ cannot be improved further. Next, I will discuss the high-frequency regimes, where it proves insufficient to consider the main order term of the resolvent asymptotics only. Physically, these are related to the transitional regimes from frequencies characteristic of Rayleigh scattering (i.e., where the wavelength in the inclusion is much larger than its size) to the ones of Mie scattering (where this wavelength is comparable to the size of the inclusion). Here I will argue that an effective model akin to that of a zero-range perturbation with internal structure (introduced in 1980s by Boris Pavlov) takes place of the effective model of [1].

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References

[1] Cherednichenko, K., Kiselev, A., Silva, L., "Operator-norm resolvent asymptotic analysis of continuous media with low-index inclusions", arXiv: 2010.13318.

[2] H. Ammari, H. Kang, K. Kim, H. Lee, 2013. Strong convergence of the solutions of the linear elasticity and uniformity of asymptotic expansions in the presence of small inclusions. J.Differential Equations, 254, 4446–4464.

[3] K. D. Cherednichenko, Yu. Ershova, and A. V. Kiselev, Effective behaviour of critical-contrast PDEs: micro-resonances, frequency conversion, and time dispersive properties. I. Communications in Mathematical Physics 375: 1833–1884, 2020.

Attenuation of acoustic waves in a composite rod

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Keywords: acoustic wave, reflection, refraction, Knott – Zoeppritz equation, impedance.

Attenuation of acoustic waves propagating in composite rods consisting of alternating materials with contrast acoustic impedances is studied by the FEA. The research is aimed at developing the vibration and seismic isolating systems capable of substantially decreasing vibration magnitudes but preserving mechanical energy; the need in devices with such a capability ranges from applications in micro- and nanoelectronics to the seismic isolation of the critically important structures, where no heat generation at the energy dissipation is allowed.

The research is based on the acoustic reflection-refraction analysis at the interfaces between acoustically contrast materials in a contact. All the rod waves are assumed to have normal incidence at the corresponding interfaces. The analysis rests upon the Knott – Zoeppritz equations [1, 2] for the reflection/refraction coefficients:

$$k_r = \frac{Z_2 - Z_1}{Z_1 + Z_2}, \qquad k_t = \frac{2Z_1}{Z_1 + Z_2},$$

where k_r, k_t are respectively coefficients of reflection and transmission (refraction); Z_1, Z_2 are the corresponding acoustic impedances; the first medium is associated with the incident wave and the second one with the refracted wave. For the considered acoustic waves the impedances take the form [1] $Z_k = c_k \rho_k$, $k = 1,2$, herein, c_k is the rod wave velocity, and ρ_k is the material density.

The analysis reveals (i) ability to substantially decrease acceleration, velocity and displacement magnitudes of the propagating wave comparing with the homogeneous rod by multiple reflection/refraction at the interfaces; (ii) preserving mechanical energy by eliminating the inelastic effects, and thus, avoiding the heat production at the wave propagation; (iii) achieving any desired acceleration, velocity and displacement magnitude reduction ratio by placing the adequate number of the acoustic interfaces.

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References

[1] Aki, K., Richards, P.G.., "*Quantitative Seismology*", Vol. 1. University Science Books: Mill Valley, California (2002).

[2] Kravtsov, A.V., Kuznetsov, S.V., Sekerzh-Zen'kovich, S.Y., "Finite element models in Lamb's problem", *Mech. Solids*, 46, 952–959 (2011).

Thermal conductivity of solids with coalescing spherical pores

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Keywords: overall conductivity, toroidal coordinates, Mehler–Fock transform.

The overall thermal properties of media containing insulating cylindrical inhomogeneities has been addressed recently in [1] making reference to a 2D layout. There, the cross section of the fibers is formed by two intersecting circles to simulate a variety of nonconductive fibers (e.g. electrospinned polystyrene fibers). On the other hand, intersecting circles is a relevant layout to assess the physical properties of a variety of porous materials (e.g. Gasar metals) during the processes of pore coalescence and growth.

In this work we extend the analysis addressed in [1] to a 3D framework by considering the effect of insulating inhomogeneities having the shape of intersecting spheres. The analysis aims at assessing the second-order resistivity contribution tensor $\boldsymbol{R}$

$$\Delta(\nabla T) = \frac{V_*}{V} \boldsymbol{R} \cdot \boldsymbol{q}, \tag{1}$$

which provides the corrective temperature gradient induced by the volume V_* of the inhomogeneity over the reference volume V of the background material subjected the a remotely applied heat flux $\boldsymbol{q}$. Owing to the geometric setting, reference is made to toroidal coordinates and Mehler–Fock transforms are used to represents the perturbation temperature field due to the inhomogeneity [2]. As remarked in [3] for coalescing spheres having the same diameter, the components of tensor $\boldsymbol{R}$ display a non-monotonic trend varying the distance between their centers. In particular, unlike what is observed for spheroids, the stationary values of the components of tensor $\boldsymbol{R}$ occur when spherical pores are slightly intersecting.

References

[1] Lanzoni, L., Radi, E. and Sevostianov, I., "Effect of cylindrical fibers, with cross-sections formed by two circular arcs, on the overall conductivity of a composite", International Journal of Solids and Structures, 138, 264-276, (2018).

[2] Morse, P.M., Feshbach, H., 1953. Methods of theoretical physics-I, McGraw-Hill, New York, USA.

[3] Lanzoni, L., Radi, E. and Sevostianov, I., "Effect of spherical pores coalescence on the overall conductivity of a material", Mechanics of Materials, 148, 103463, (2020).

Manipulation of deformation and bandgaps in soft porous phononic crystals by integrating with rigid inclusions

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Keywords: soft porous phononic crystal, bandgap, inclusion

Phononic crystals (PCs) have attracted intensive research interests from investigators in different disciplines including physics, mechanics, materials, devices, etc. for almost 30 years. The particular band structures in PCs make them perfect candidates for many novel and useful acoustic devices. In this paper, two-dimensional soft porous periodic structures inserted with patterned hard inclusions are considered. The finite element method is employed to study the effects of those rigid inclusions on the deformation, the buckling mode, the post-buckling deformation, and the band structure in the soft PCs subject to a large deformation. It is found that both the number and the distributive pattern of the inclusions play a significant role in affecting the bandgap characteristics. A new method approach to tuning the bandgaps is thus proposed by adjusting the filling pattern of the inclusions along with dramatically deforming the structures. Compared with the soft PCs without inclusions, the sensitivity of the post-buckling deformation to the initial geometric imperfections is also significantly reduced for the soft PCs with inclusions. Therefore, the post-buckling deformation could develop robustly, and the bandgaps can be tuned in a versatile and reversible way. Experiments are also conducted to confirm the simulation results.

References

[1] Li, J., Wang, Y.T., Chen, W.Q., Wang, Y.S., Bao, R.H., “Harnessing inclusions to tune post-buckling deformation and bandgaps of soft porous periodic structures”. *Journal of Sound and Vibration*, 459, 11848 (2019).

[2] Gao, N., Li, J., Bao, R.H., Chen, W.Q., “Harnessing uniaxial tension to tune Poisson’s ratio and wave propagation in soft porous phononic crystals: An experimental study”, *Soft Matter*, 15, 2921-2927 (2019).

Green’s functions in discrete flexural and elastic systems

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Keywords: Green’s functions, flexural waves, anisotropy.

On elastic lattices, flexural waves and torsional waves are often studied independently. In this talk, the coupling of flexural and torsional motion for out-of-plane waves in a 2D square lattice of beams will be discussed. It will be shown that altering the magnitude of the rotational inertia at the lattice junctions, and the magnitude of the torsional stiffness in the beams allows significant control over the dispersion properties of the lattice. The Green’s function is derived analytically and evaluated numerically; the analysis is supported by an independent finite element model. Examples of evanescent and propagative wave modes will be discussed and it will be shown how applied forces, applied moments, and combinations thereof can be used to induce dynamic anisotropy, including asymmetric anisotropy. This work has applications in many areas where the control of wave propagation through elastic structures is of interest, including energy harvesting, energy dissipation and metamaterial design.

Exact and approximate formulas for the effective properties of 2D high contrast composites

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Keywords: Random composites, effective constants, Riemann-Hilbert problem.

Two-dimensional elastic and conductivity problems for regular and random composites with non-overlapping circular inclusions are solved by functional equations associated to Schwarz's alternating method. The wrong opinion about these problems that "complete solutions are virtually impossible" dominates in literature. The first exact formula for the effective conductivity of square array of holes was derived in 1998. This approach was outlined in the books [1, 2] where analytical approximate formulas for the effective constants were summarized for random composites. The method is based on the exact solution to the Riemann-Hilbert and R-linear problems for a circular multiply-connected domain. It should be stressed that *exact solution* means *exact formula*, when the desired effective constants are written in the left side and the input data, centers and radii of inclusions, the constants of components are explicitly presented in the right side [3]. Such a deterministic (exact or approximate) formula yields the result for a given set of parameters in symbolic form. The passage to random composites is based on the direct computation of the mathematical expectation over a class of random composites. The obtained results diverge with some previous approximations (self-consistent approaches) where conditionally divergent series and sums were used without proper analysis. The special attention is paid to high contrast composites when the percolation effects may occur. In this case, the obtained formulas are renormalized to extract the corresponding singularities.

References

[1] Gluzman, S., Mityushev, V., Nawalaniec, W., Computational Analysis of Structured Media, Academic Press, Elsevier (2018).

[2] Drygas, P., Gluzman, S., Mityushev, V., Nawalaniec, W., Applied Analysis of Composite Media. Analytical and Computational Results for Materials Scientists and Engineers, Elsevier (2020).

[3] Andrianov, I., Mityushev, V., Exact and "Exact" Formulae in the Theory of Composites, Modern Problems in Applied Analysis, eds: P. Drygas, S. Rogosin, Birkhauzer, Springer International Publishing AG, Cham, 15–34 (2018).

Approximations for vibration modes of membranes with clusters of small inertial inclusions

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Keywords: mesoscale approximations, membranes, inertial clusters.

We discuss a formal asymptotic approach used to analyse vibrations supported by finite elastic membranes containing clusters of small inertial inclusions [1]. The procedure developed utilises the method of mesoscale approximations [2] and is shown to provide accurate predictions for low-frequency eigenvalues and the associated eigenmodes of such composites. In particular, we show the eigenvalues can be traced from the degeneracies of a homogeneous linear algebraic system that contains information about the inertial cluster's properties. This linear system arises naturally in approximating the eigenmodes of the membrane and is essential in ensuring the approximations satisfy the governing equations to a high order of accuracy. Numerical illustrations are also provided that demonstrate the efficiency of the asymptotic approach.

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References

[1] Maz'ya, V., Movchan, A.B., Nieves, M.J.: On meso-scale approximations for vibrations of membranes with lower-dimensional clusters of inertial inclusions, Algebra i Analiz, no. 3, 219-237 (2020).

[2] Maz'ya, V., Movchan, A., Nieves, M.: Green's Kernels and Meso-scale Approximations in Perforated Domains, Lecture Notes in Math. 2077, Springer (2013).

Dispersive elactic waves in high-contrast layered structures studied by FEM analysis

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Keywords: Layered composites, high-contrast properties, elastic waves, Rayleigh-Lamb dispersion, cut-off frequency, modal FEM analysis.

In the last decades, multi-layered plates have been extensively studied due to the significant increase in applications of high-tech composite structures in various industries such as civil engineering, modern aerospace and automotive. In specific applications, sandwich-like structures may exhibit high contrast of geometrical and physical properties of layers. This poses a great challenge for the adequate analysis of the dynamic response, and motivate the development of an appropriate methodology for the accurate interpretation of the vibration spectra.

The dynamic behavior of multi-layered plate-like structure is characterized by Lamb waves, guided by free upper and lower surfaces, and taking into account reflection and refraction between the layers and scattering at the upper and lower traction-free planes. A new asymptotic approach for deriving dispersion equations in a relatively simple form has been developed by Kaplunov et al. [1], and the corresponding comprehensive dynamic analysis for strongly inhomogeneous three-layered plate has been performed as well. Such approach, based on the guided waves propagation, provides a strong analytical background which may assist in the optimal structural design, as well as development of the damage detection strategies.

In our research, we present an engineering numerical methodology to study the Lamb waves in a thin, high-contrast three-layer composite plate with contrast properties of the layers using the commercial finite element method (FEM) software. The fundamental antisymmetric mode A_0, which carries most of the energy and dominates over the symmetric one, and the first harmonic antisymmetric mode A_1, which determines cut-off frequency, are analyzed. The obtained results are compared with the iterative numerical solution of the Rayleigh-Lamb dispersion equation for the corresponding modes. It is shown that the complexity of the dispersion phenomena in the multi-layered plates can be resolved accurately by post-processing the FEM modal analysis results. The advantage of the proposed methodology consists in the convenience and availability of its application in wide engineering practice.

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References

[1] Kaplunov, J., Prikazchikov, D.A., Prikazchikova, L.A., " Dispersion of elastic waves in a strongly inhomogeneous three-layered plate", *Int. J. Solid Struct.*, 113–114, 169–179 (2017)

Reduced asymptotically consistent model of microstructured plates

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Keywords: asymptotic method, Microstructured plate, Couple stress, Rayleigh-Lamb waves

Several beam and plate models have been recently developed in the literature to accommodate for size-dependence. These are usually obtained starting from a generalized continuum theory (such as the couple-stress, strain-gradient or non-local theory or their modifications) and then deducing the governing equations through Hamilton's principle and ingenuous kinematical assumptions. This approach, originated by Kirchhoff, usually fails to reproduce the dispersion features of the equivalent 3D theory. Besides, it produces a variety of models, in dependence of the different assumptions, such as Kirchhoff's or Mindlin's. In contrast, in this talk we proceed by asymptotic reduction: moving from the couple-stress linear theory of elasticity with micro-inertia, we deduce new models for elongation and flexural deformation of microstructured plates. We show that microstructure especially affects inertia terms, which can be hardly captured by a-priori kinematical assumptions.

References

[1] Erbaş, B., Kaplunov, J., Nobili, A., Kılıç, G., 2018. Dispersion of elastic waves in a layer interacting with a Winkler foundation. The Journal of the Acoustical Society of America 144, 2918–2925.

[2] Nobili, A., Radi, E., Signorini, C., 2020. A new Rayleigh-like wave in guided propagation of antiplane waves in couple stress materials. Proceedings of the Royal Society A 476, 20190822.

[3] Nobili, A. "Asymptotically consistent size-dependent plate models based on the couple-stress theory with micro-inertia." European Journal of Mechanics-A/Solids 89 (2021): 104316.

Nonlocal diffusion in higher-order thermal lattices

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Keywords: Heat equation, Fourier's law, Lattice, Nonlocality, Higher-order thermal lattice, Short range and long range interactions

The present paper deals with heat conduction in one-dimensional thermal lattices, accounting for some short range and long range thermal interactions. We firstly derive exact solution for the evolution problem of a two-neighbour interaction Fourier lattice. An exact solution based on the method of separation of variables and the use of trigonometric series, for given initial and boundary conditions, is obtained. The solution is then compared to the classical continuous Fourier solution and to a continuous nonlocal solution corrected by some length scale effects. The nonlocal solution is a differential nonlocal Fourier heat model. The length scale parameter for this model is calibrated by applying a continualization method to the mixed difference differential thermal lattice equations. An error analysis is performed in order to verify the efficiency of the local and nonlocal approximations. Finally, we investigate the behaviour of the general *p*-order thermal lattice (which accounts for the *p*-order neighbouring), both in the discrete and in its continuous nonlocal approximations. The length scale of this discrete diffusion problem is calibrated, whatever the nature of the short range or long range thermal interactions.

High contrast asymptotic expansion for fluid-structure interaction

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Keywords: linear elasticity, thin domain, Stokes equations, high contrast asymptotic expansion, dimension reduction.

High contrast asymptotic analysis is a tool to study contact problems for materials with strongly different physical constants, such as stiffness or conductivity ([1-5], [7-12]). The talk presents applications of this method to the non-stationary fluid-structure interaction problems, where the parameter of high contrast is the ratio of the product of the stiffness of elastic material by the characteristic time of the process to the dynamic viscosity. The fluid-structure interaction in the case of comparable properties of the fluid and elastic material was considered in [6]. We consider the case when the elastic part is thin (with respect to its length) while the fluid part is described by the non-stationary Stokes equations and can be thick or thin [7], [9-11].

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References

[1] Cherednichenko K., Cherdantsev M., "Two-scale Γ-convergence of integral functionals and its application to homogenisation of nonlinear high-contrast periodic composites", *Arch. Ration. Mech. Anal.,* 204, 445-478 (2012).

[2] Cherednichenko K.D., Smyshlyaev V.P., Zhikov V.V., "Non-local homogenized limits for composite media with highly anisotropic periodic fibres". *Proc. R. Soc. Edinburgh A – Math.*, 136, 87-114 (2006).

[3] Dai H.H., J.Kaplunov J., Prikazchikov D.A., "A long wave model for the surface elastic wave in a coated half-space", *Proc. R. Soc. A,* 466, 2122, 3097-3116 (2010)

[4] Figotin A., Kuchment P., "Spectral properties of classical waves in high-contrast periodic media", *SIAM J. Appl. Math.*, 58, 683 – 702 (1998)

[5] Lions J.-L., *"Perturbations singuliéres dans les problèmes aux limites et contrôle optimal", Lect. Notes in Math.,* 323, Berlin, 1973.

[6] Mikelić A., Guidoboni G., Čanić S.,«Fluid-structure interaction in a pre-stressed tube with thick elastic walls. I. The stationary Stokes problem", *Netw. Heterog. Media,* 2, 397 – 423 (2007).

[7] Orlik J., Panasenko G., Stavre R., "Asymptotic analysis of a viscous fluid layer separated by a thin stiff stratified elastic plate", *Applicable Analysis*, 100,3, 589-629 (2021)

[8] Panasenko G.P. "Asymptotics of solutions and eigenvalues of elliptic equations with strongly variable coefficients", *Doklady Akademii Nauk SSSR.*, 252,6,1320-1325 (1980).

[9] Panasenko G., Stavre R., "Viscous fluid – thin elastic plate interaction: asymptotic analysis with respect to the rigidity and density of the plate", *Appl. Math. Optim.,* 81, 1, 141-194 (2020)

[10] Panasenko G., Stavre R., "Viscous fluid – thin cylindrical elastic tube interaction: asymptotic analysis on contrasting properties", *Applicable Analysis*, 98, 1 and 2, 162-216 (2019)

11] Panasenko G., Stavre R., "Three dimensional asymptotic analysis of an axisymmetric flow in a thin tube with thin stiff elastic wall", *J. Math. Fluid Mech.*, 22, 20, 35pp. (2020)

[12] Simonenko I.B., "Problems of electrostatics in an inhomogeneous medium. The case of a thin dielectric with a large dielectric constant. I", *Differ. Uravn.*, **10**, 2, 301–309 (1974)

Metaconcrete: Engineered aggregates for enhanced dynamic performance

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Recent progress in the field of metamaterials science has led to a range of novel composites which display unusual properties when interacting with electromagnetic, acoustic, and elastic waves. Metaconcrete is a new structural metamaterial with enhanced wave attenuation properties for dynamic loading applications. In this new composite material the standard stone and gravel aggregates of regular concrete are replaced with spherical engineered inclusions. Each metaconcrete aggregate has a layered structure, consisting of a heavy core and a thin compliant outer coating. This structure allows for resonance at or near the eigenfrequencies of the inclusions, and the aggregates can be tuned so that resonant oscillations will be activated by particular frequencies of an applied dynamic loading. The activation of resonance within the aggregates causes the overall system to exhibit negative effective mass, which leads to attenuation of the applied wave motion.

The first numerical investigations, considering a finite element slab model containing a periodic array of aggregates, confirmed the expected attenuation behavior of metaconcrete under a variety of different loading conditions. The various analyses discussed in our works provide the theoretical and numerical background necessary for the informed design and improvement of metaconcrete aggregates for dynamic loading applications, such as blast shielding, impact protection, and seismic mitigation. Subsequent works involved the preparation of several samples of metaconcrete with different shapes, inclusions, and sizes. Every experimental study contributed to elucidate different aspects and peculiarities of metaconcrete.

The work has been developed in collaboration with Stephanie J. Mitchell (Caltech), Michael Ortiz (Caltech), and Deborah Briccola (Polimi), and the enthusiastic support of MS and PhD students.

References

[1] D. Briccola, M. Cuni, A. De Juli, M. Ortiz, and A. Pandolfi. Experimental validation of the attenuation properties in the sonic range of metaconcrete containing two types of resonant inclusions. Experimental Mechanics, online first:1-18, 2021.

[2] D. Briccola and A. Pandolfi. Analysis on the dynamic wave attenuation properties of metaconcrete considering a quasi-random arrangement of inclusions. Frontiers in Materials, 7:615189:1-14, 2021.

[3] D. Briccola, M. Tomasin, T. Netti, and A. Pandolfi. The influence of a lattice-like pattern of inclusions on the attenuation properties of metaconcrete. Frontiers in Materials, 6:1-11, 2019.

[4] D. Briccola, M. Ortiz, and A. Pandolfi. Experimental validation of metaconcrete blast mitigation properties. Journal of Applied Mechanics, 84:031001-6, 2017.

[5] S. J. Mitchell, A. Pandolfi, and M. Ortiz. Effect of brittle fracture in a metaconcrete slab under shock loading. Journal of Engineering Mechanics, ASCE, 142(4):040160, 2016.

[6] S. J. Mitchell, A. Pandolfi, and M. Ortiz. Investigation of elastic wave transmission in a metaconcrete slab. Mechanics of Materials, 91:295-303, 2015.

[7] S. J. Mitchell, A. Pandolfi, and M. Ortiz. Metaconcrete: designed aggregates to enhance dynamic performance. Journal of the Mechanics and Physics of Solids, 65:69-81, 2014.

The prediction of defect strengths in nematic shells.

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Keywords: Topology, Liquid Crystals.

Nematic liquid crystals are an intermediate phase of matter between crystalline solid and isotropic liquid states, whose molecules closely resemble rods [2]. A nematic shell is a thin film of nematic liquid crystal coating a curved substrate, which is a modelled as a smooth two-dimensional manifold which is embedded into three-dimensions [4]. Liquid crystals can also possess defects in their patterns, localised discontinuities which can occur as either isolated points or as lines [5]. A natural way to model the preferred direction of orientation would be a vector field with a corresponding energy functional, in the same manner to the one-constant Oseen-Frank model [3].

In this model the presence of defects results in a divergence of the energy density, thus we regularise by excluding small regions about each of the defects. There are topological laws that vector fields, and by extension our liquid crystal, must obey such as the Poincaré–Hopf theorem [1]; however, this theorem is only applicable to closed manifolds.

I shall present a generalisation of the Poincaré–Hopf theorem, which accounts for manifolds with boundaries and boundary defects. Then I shall utilise the divergence of the energy density to heuristically predict the defect strengths given only the geometrical parameters.

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References

[1] M. P. Carmo. Differential Geometry of Curves and Surfaces. Prentice-Hall Inc., 1976.

[2] P. de Gennes. The physics of Liquid Crystals. The international series of monographs of Physics. Clarendon Press, 1974.

[3] F.C.Frank. I. liquid crystals. on the theory of liquid crystals. Discussions of the Faraday Society, 25:19–25, 1958.

[4] G. Napoli and L. Vergori. Surface free energies for nematic shells. Physical Review, 85, 2012.

[5] E. Virga. Variational Theories for Liquid Crystals, volume 8 of Applied Mathematics and Mathematical Computation. Chapman and Hall, 1st edition, 5 1995.

Stress concentration due to a hole in a body with two family of fibers

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We study the stress concentration factor due to circular hole in a nonlinearly elastic plate reinforced by two families of fibers. We restrict our attention to the class of materials wherein the fibers are mechanically equivalent, rendering the body orthotropically symmetric with respect to the planes whose normals are along the angular bisectors of the angles between the fibers. We study the problem of equibiaxial, uniaxial and pure shear deformations of the body whose fibers are symmetrically oriented with respect to the Cartesian coordinate axes. These deformations indicate strong influence of the fiber angle on the stress concentration factor.

Elastic instabilities in heterogeneous materials under finite strains

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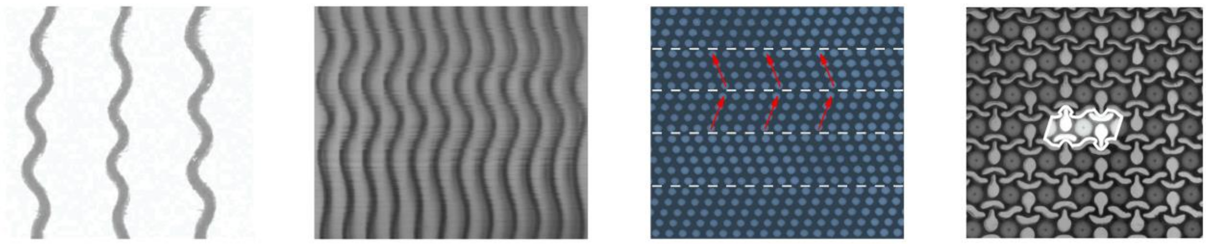
Keywords: fiber composites, soft materials, instabilities.

The elastic instability phenomenon plays an important role in pattern formations in soft biological systems. The phenomenon also has been actively used to design new (meta-) materials with switchable microstructures, properties, and functions [1].

Here, we investigate the elastic instability phenomenon in soft heterogeneous materials. These deformable composites typically combine soft deformable matrix and stiffer phase (such as fibers or inclusions). Figure 1 shows the experimentally observed wavy patterns forming in (a) soft 3D printed laminates [2] and (b) 3D-fiber composites [3], (c) twining pattern in particulate composites [4], and (d) auxetic microstructure transformations in inclusion-matrix-void soft systems [5].



(a) Laminate (b) 3D fiber composite (c) Particulate composite (d) Multiphase composite

Figure 1. Instability-induced wavy patterns forming in (a) soft 3D printed laminates [2] and (b) 3D-fiber composites [3]; (c) twining pattern in particulate composites [4], and (d) auxetic microstructure transformations in inclusion-matrix-void soft systems [5].

References

[1] Li, J., Arora, N., S Rudykh, S., "Elastic instabilities, microstructure transformations, and pattern formations in soft materials", *Current Opinion in Solid State and Materials Science* 25, 100898 (2021)

[2] Slesarenko. V., Rudykh, S., "Harnessing viscoelasticity and instabilities for tuning wavy patterns in soft layered composites. " *Soft Matter* 12, 3677-3682 (2016)

[3] Li, J., Slesarenko, V., Galich, P.I., Rudykh, S., "Instabilities and pattern formations in 3D-printed deformable fiber composites. " *Compos Part B Eng.*, 148:114-122 (2018)

[4] Li, J., Pallicity, T.D., Slesarenko, V., Goshkoderia, A., Rudykh, S., "Domain Formations and Pattern Transitions via Instabilities in Soft Heterogeneous Materials. " *Adv Mater.*, 31, 1807309 (2019).

[5] Li, J., Slesarenko, V., Rudykh, S., "Auxetic multiphase soft composite material design through instabilities with application for acoustic metamaterials." *Soft Matter*, 14, 6171-6180 (2018).

Nonlinear elasticity may be helpful in elastography?

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Keywords: Elastography, Nonlinear waves, Weakly nonlinear materials

Elastography is a medical imaging modality to produce elastograms i.e. maps of the elastic properties and stiffness of soft tissues. This is a quantitative version of the classical palpation practice to feel the stiffness of a patient's tissues by using hands. It is well known that, for example, cancerous tumors will often be harder than the surrounding tissue.

The actual methods of elastographic investigation are based on linear wave propagation. It is therefore interesting to explore if it is possible to improve this technique using nonlinear wave propagation. To this end we first explain why nonlinear elasticity may improve the elastograms and then we investigate which mathematical methods may be used to exploit the propagation of nonlinear waves.

Asymptotic analysis of the influence of the end conditions on the lowest vibrations of composite beams

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Keywords: composite beam, low-frequency vibration, asymptotic analysis.

Composite materials have become an important research field in various fields of civil and mechanical engineering with the latest technological developments. Sandwich structures, laminated glass beams and plates, smart periodic structures and photovoltaic panels may be given as examples for composite structures used in various branches of engineering. We also mention the multiparametric asymptotic approach for strongly inhomogeneous structures i.e. rods, plates, beams, etc., which have been actively studied in recent years.

In this work, the effects of end conditions on the lowest vibration modes of strongly inhomogeneous beams are analyzed. An asymptotic procedure based on a small parameter arising from the high contrast between the components of the beamis employed to obtain eigenfrequencies and eigenforms of the considered problems. The proposed model is employed for two and three-component composite beams with different end conditions. According to the obtained results, the effects of the end conditions are reviewed. Finally, comparisons of exact and approximate solutions are presented, demonstrating the validity of the proposed approach.

References

[1] Milton, G.W., "The Theory of Composites", Cambridge University Press (2002).

[2] Aßmus, M., Naumenko, K., Altenbach, H., "Mechanical behaviour of photovoltaic composite structures: Influence of geometric dimensions and material properties on the eigenfrequencies of mechanical vibrations", Composites Communications, 6, 59-62 (2017).

[3] Martin, T.P., Layman, C.N., Moore K.M., Orris, G.J., "Elastic shells with high-contrast material properties as acoustic metamaterial components", Phys. Rev. B, 85, 1611,03 (2012).

[4] Schulze, S.H., Pander, M., Naumenko, K., Altenbach, H.,"Analysis of laminated glass beams for photovoltaic applications", Int. J. Solids Struct., 49, 15, 2027-2036 (2012).

[5] Ruzzene,M., Baz, A., "Attenuation and localization of wave propagation in periodic rods using shape memory inserts", Smart Mater. Struct., 9(6), 805-816 (2000).

[6] Kaplunov, J., Prikazchikov, D., Sergushova, O., "Multi-parametric analysis of the lowest natural frequencies of strongly inhomogeneous elastic rods", J. Sound Vib., 366, 264–276 (2016).

[7] Şahin, O., Erbaş, B., Kaplunov, J., Savšek, T., "The lowest vibration modes of an elastic beam composed of alternating stiff and soft components", Archive of Applied Mechanics, 90(2), 339-352 (2020).

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A Cosserat model of elastic solids reinforced by a family of curved and twisted fibres

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Keywords: Cosserat elasticity, fibre-reinforced solids, material symmetry.

We outline a Cosserat model for fibre-reinforced solids in which the fibers are modelled as continuously distributed spatial Kirchhoff rods with intrinsic flexural, torsional and extensional elasticity. The basic kinematical variables are a deformation field and a rotation field that describes the local fibre orientation. Constraints on these fields are introduced to account for the materiality of the fibers with respect to the underlying continuum deformation, with the associated Lagrange multipliers interpreted as transverse shear tractions acting on the fiber cross sections. The theory is illustrated via simple examples involving finite deformation.

Acknowledgments

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References

[1] Steigmann, D.J., "Theory of elastic solids with fibres resistant to extension, flexure and twist", *Int. J. Non-linear Mech.*. **47**, 734-742 (2012).

[2] Shirani, M., Steigmann, D.J., "A Cosserat model of elastic solids reinforced by a family of curved and twisted fibres", *Symmetry*, **12**, 1133 (2020).

Understanding the strength of bioinspired composites

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Keywords: Strength, bioinspired, composite.

Remarkable mechanical properties of biocomposites (bone, teeth, shell, antler etc.) are usually attributed to their special design where staggered mineral platelets are embedded in a protein matrix. Because of the high aspect ratio of the platelet, the soft protein deforms in the shear mode predominantly providing the linkage for the hard inclusions. Mimicking Nature one might design materials with a similar architecture.

By employing a micromechanical analysis, we study in the present work the strength of a bio-inspired composite in which hard platelets are embedded in a soft matrix made of the vulcanized natural rubber. We perform simulations of uniaxial tension of the composite material based on a continuum mechanics formulation and the high-fidelity generalized method of cells. The use of the energy limiters in the constitutive model for rubber at finite strains allows us to model failure and arrive at the overall strength of the composite.

We find that the overall strength of the composite depends on the deformation and failure of soft matrix in tension and shear. Moreover, we find that the strength of the composite cannot exceed the strength of the matrix. The latter observation is noteworthy because it is qualitatively different from the previous experimental results with biocomposites which show a dramatic (ten times) increase of the strength of the material as compared to the strengths of its constituents. We illustrate these analytical and numerical findings by our experiments on 3D printed composite materials [1].

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References

[1] Slesarenko, V., Volokh, K.Y., Aboudi, J., Rudykh, S., "Understanding the strength of bioinspired soft composites", *Int. J. Mech. Sci.*, 131-132, 171-178 (2017)

High-contrast dynamic homogenisation of periodic micro-resonances with random properties

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Keywords: High-contrast homogenisation, random micro-resonances, localisation.

Macroscopic dynamic properties of composite materials containing “micro-resonant” inclusions can be very different from those of conventional materials. Mathematically this leads to studying homogenisation of problems with a “critically” scaled high contrast, where the resulting two-scale asymptotic behaviour appears to display a number of interesting effects. For a background and twoscale analysis of general “partially-degenerating” periodic high-contrast PDE problems see [1], and for a recent application to high-contrast periodic Timoshenko elastic beam lattice materials displaying band gap-type effects see [2]. Here we discuss situations where the high-contrast inclusions display certain randomness. In simplest cases of a periodic geometry but random coefficients in the spherical inclusions, the resulting two-scale limit behaviour appears to be rather explicit and the macroscopic equations display a wave localisation i.e. a kind of trapping by the micro-resonances due to their randomness.

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References

[1] Kamotski, I.V., Smyshlyaev, V.P., ”Two-scale homogenization for a general class of high contrast PDE systems with periodic coefficients”, Applicable Analysis, 98, 64-90 (2019).

[2] Kamotski, I.V., Smyshlyaev, V.P., ”Bandgaps in two-dimensional high-contrast periodic elastic beam lattice materials”, J. Mech. Phys. Solids, 123, 292-304 (2019).

Wave propagation in fluid-filled periodic shells composed of high-contrast cells

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Keywords: Reduced-order modelling, Pass- and stop-bands, Eigenfrequencies.

We consider wave propagation in a compound shell, which consists of periodically alternating stiff curved and compliant straight elements with the same inner diameter. The shell is exposed to heavy internal loading by a compressible fluid. Our aim is to design such a 'snake-shape' insert as a filter to suppress low-frequency pressure pulsations in a fluid.

The classical periodicity stop-band effect enhanced by the high contrast between waveguide properties of a pre-stressed membrane cylindrical shell and a stiff metallic curved shell is studied using a hierarchy of analytical reduced-order models for each constituent, including classical elastic fluid-filled shell, acoustic duct, Korteveg membrane shell and spin-off models.

To identify location of pass- and stop-bands, we perform the modal analysis of free vibrations of a unit symmetric periodicity cell with boundary conditions set up in accordance with the bi-orthogonality relations for constituents of a periodic shell. As has been shown in [1-2], conventional Floquet modelling of wave propagation in an infinite periodic waveguide predicts boundaries between pass- and stop-bands, which coincide with the eigenfrequencies of the periodicity cell with these boundary conditions. The attention is focused at sensitivities of location and widths of stop-bands to pre-stress and length of membrane segments.

Then we calculate Insertion Losses introduced by this 'snake-shape' insert, which consists of a small number of periodicity cells, and compare attenuation levels inside and outside stop-bands for varying number of cells. The next step in this on-going research project concerns experimental work, and now the laboratory setup is under reconstruction and modernization. We hope to obtain experimental data for verification of our theoretical and numerical predictions by the time Colloquium commences.

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References

[1] Sorokin, S.V., Gautier, F., Pelat, A., "A hierarchy of models of axisymmetric wave propagation in a fluid-filled periodic cylindrical shell composed of high-contrast cells", *Mechanical Systems and Signal Processing* 136 106487 (2020).

[2] Ledet, L. S., and Sorokin, S.V., "On the application of bi-orthogonality relations for analysis of linear dynamic systems", *Journal of Sound and Vibration* 465 115020 (2019).

Asymptotic homogenization method applied to three-dimensional micropolar heterogeneous media

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Keywords: Micropolar media, Asymptotic homogenization method, Centro-symmetric Cosserat laminated composite.

This work deals with the application of the two-scale asymptotic homogenization method (AHM) to three-dimensional heterogeneous micropolar (Cosserat) media with periodic structure. Micropolar elasticity is a generalization of the classic theory of elasticity which incorporates three degrees of freedom (DoF) to describe the local reorientation of the microstructure (microrotations) and three ones related to the displacement at the macroscale. The AHM is based on the consideration of two length scales associated to the microscopic and macroscopic phenomena. The AHM scheme for a 3D heterogeneous micropolar material starts from the statement of the problem based on the microscopic-macroscopic description. Thus, the corresponding local problems, the homogenized problem and the effective coefficients are obtained through asymptotic expansions for displacements and microrotations.

In particular, micropolar bi-laminated composites which satisfied the centro-symmetric postulate are studied. Two cases are considered: a) a bi-laminated composite where each layer has cubic symmetry and b) a bi-laminated composite with isotropic constituents. In both cases, the corresponding local problems are solved and the analytical expressions of the effective properties are reported. Numerical values of the stiffness, torque, Poisson's ratio, Young's modulus, twist Poisson's ratio and torsional modulus are computed and presented for different volume fractions of the constituents.

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